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# $K \rightarrow \pi \nu \bar{\nu}$ in and beyond the SM

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Technion

# Rare Kaon decays

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Many interesting modes beside  $K \rightarrow \pi \nu \bar{\nu}$

- LFV modes,  $K_L \rightarrow \mu e$ ,  $K \rightarrow \pi \mu e$ , ...
- L violation modes,  $K^+ \rightarrow \pi^- \ell^+ \ell^+$ , ...
- Rare FCNC,  $K_L \rightarrow \pi^0 \ell^+ \ell^-$ , ...

Yet, I will talk about  $K \rightarrow \pi \nu \bar{\nu}$ .

# Some references

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- Littenberg, 1989
- Buchalla and Buras, 1990s
- Grossman and Nir, 1997
- Nir and Worah, 1997
- Falk and Petrov, 2001
- D'Ambrosio and Isidori, 2001
- Grossman, Isidori and Murayama, 2003

# Outline

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- $K \rightarrow \pi \nu \bar{\nu}$  in specific models
  - General considerations
  - Standard Model
  - New physics
- General properties of  $K_L \rightarrow \pi \nu \bar{\nu}$ 
  - Why is  $K_L \rightarrow \pi \nu \bar{\nu}$  CP violating?
  - What kind of CP violation?
  - Possible CP conserving contributions
- Conclusions

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# $K \rightarrow \pi \nu \bar{\nu}$ in specific models

# Why is $K \rightarrow \pi \nu \bar{\nu}$ so clean?

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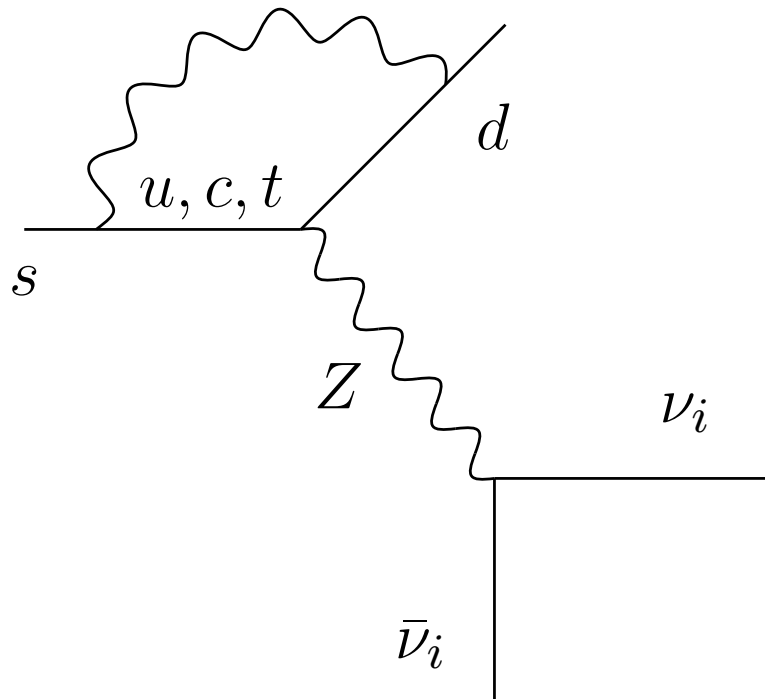
- Hadronic matrix elements  $\Leftarrow$  Isospin
- Long distance effects  $\Leftarrow$  Neutrinos

Therefore the  $K \rightarrow \pi \nu \bar{\nu}$  decays are very interesting

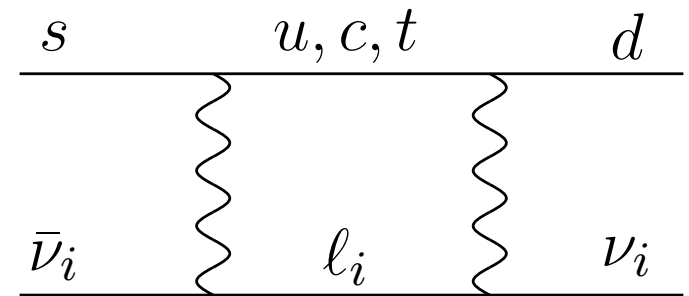
- Exclusive hadronic decays that we can calculate in the SM and its extensions
- The measured rates are sensitive to fundamental parameters



# Example: the standard model



Penguin and box diagrams



$$A(K \rightarrow \pi \nu \bar{\nu}) = \sum_{q=u,c,t} A_q$$

# $K \rightarrow \pi \nu \bar{\nu}$ in the SM

$$A(K \rightarrow \pi \nu \bar{\nu}) = \sum_{q=u,c,t} A_q$$

where

$$A_q \sim m_q^2 V_{qs}^* V_{qd} \sim \begin{cases} \Lambda_{QCD}^2 \lambda & up \\ m_c^2 (\lambda + i\lambda^5) & charm \\ m_t^2 (\lambda^5 + i\lambda^5) & top \end{cases}$$

- Hard GIM, negligible LD effects
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ : top dominant, charm is important
- $K_L \rightarrow \pi \nu \bar{\nu}$ : CP violating, almost pure top

# $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the SM

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The fundamental Wolfenstein parameters:  $\lambda$ ,  $A$ ,  $\rho$  and  $\eta$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = C_+ |A|^4 \times [\eta^2 + (\rho - \rho_c)^2] = O(10^{-10})$$

- $C_+$  is known
- $\rho_c$  includes the charm effect, with small theoretical errors
- The largest error in extracting  $\rho$  and  $\eta$  is from  $A$

# $K_L \rightarrow \pi \nu \bar{\nu}$ in the SM

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$$\mathcal{B}(K_L \rightarrow \pi \nu \bar{\nu}) = C_L |A|^4 \times \eta^2 = O(10^{-11})$$

- CP violating decay
- Very small theoretical error in  $C_L$
- The largest error in extracting  $\rho$  and  $\eta$  is from  $A$
- In the ratio  $A$  cancels

$$\frac{\mathcal{B}(K_L \rightarrow \pi \nu \bar{\nu})}{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} = \frac{C_L}{C_+} \times \frac{\eta^2}{\eta^2 + (\rho - \rho_c)^2}$$

# New physics effects

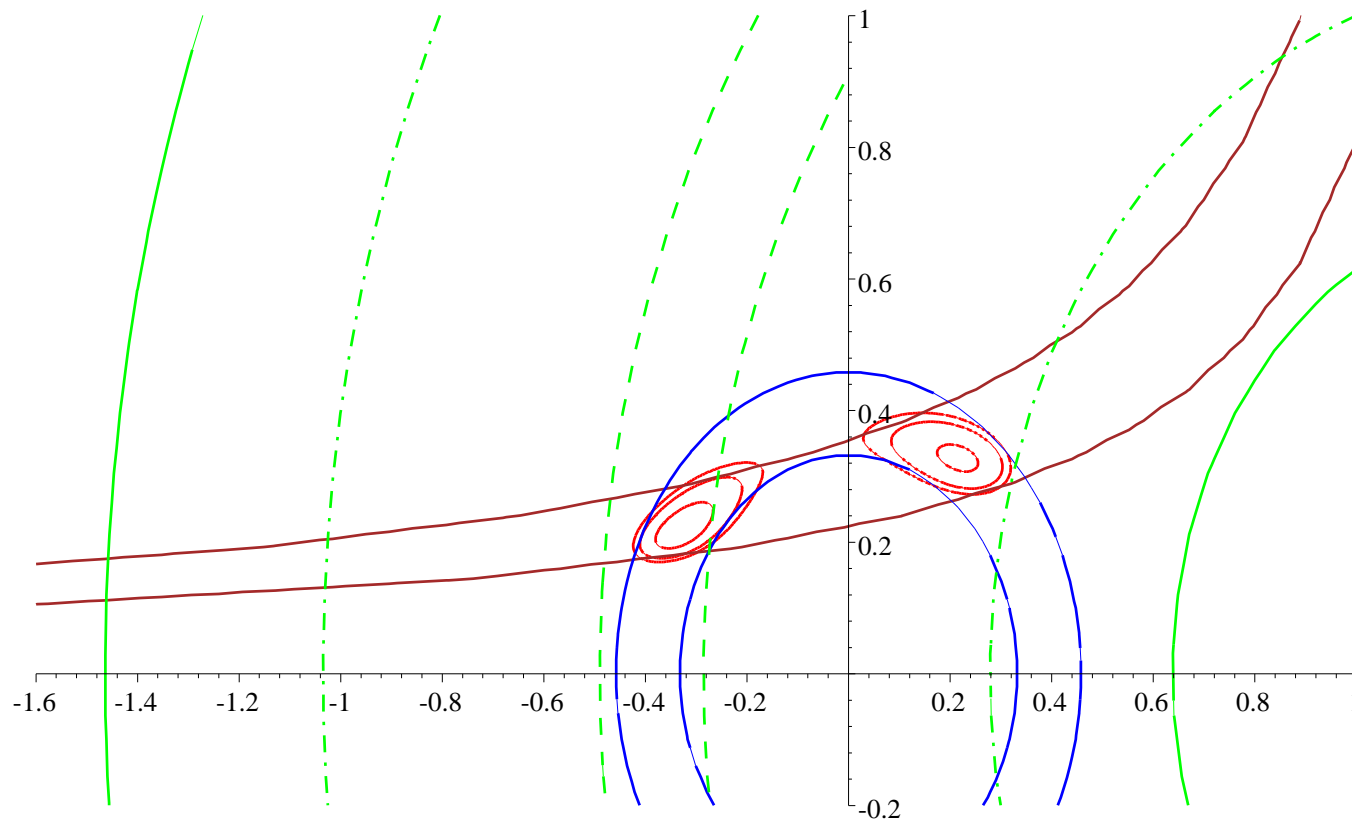
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New physics can affect the rates in many ways

- Significant new physics only in the  $B$  sector
- Minimal flavor violation: flavor violation is confined to the SM Yukawa couplings
- Significant new physics in  $K \rightarrow \pi \nu \bar{\nu}$  decays

# New physics only in the $B$ sector

G. Isidori

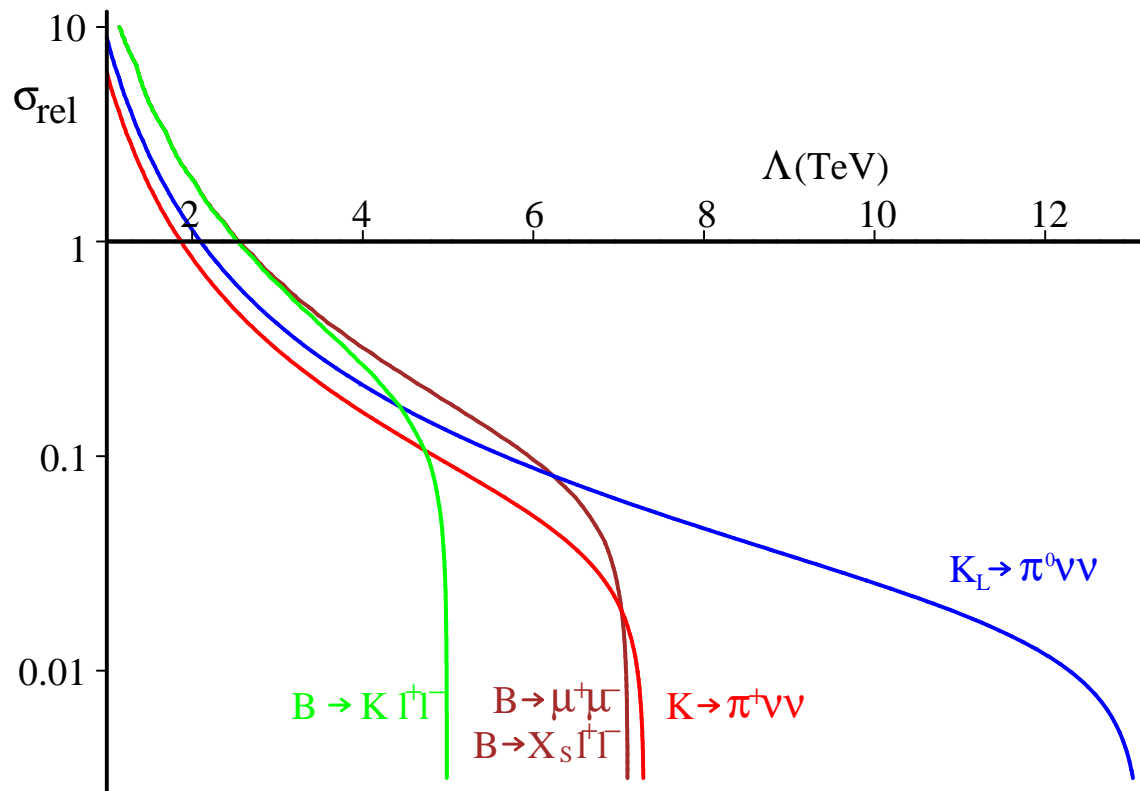


● The green lines: central value,  $1\sigma$  and 90% CL

# Minimal flavor violation

- One effective new parameter: the new physics scale
- Small effects, but  $K \rightarrow \pi \nu \bar{\nu}$  have very high sensitivity

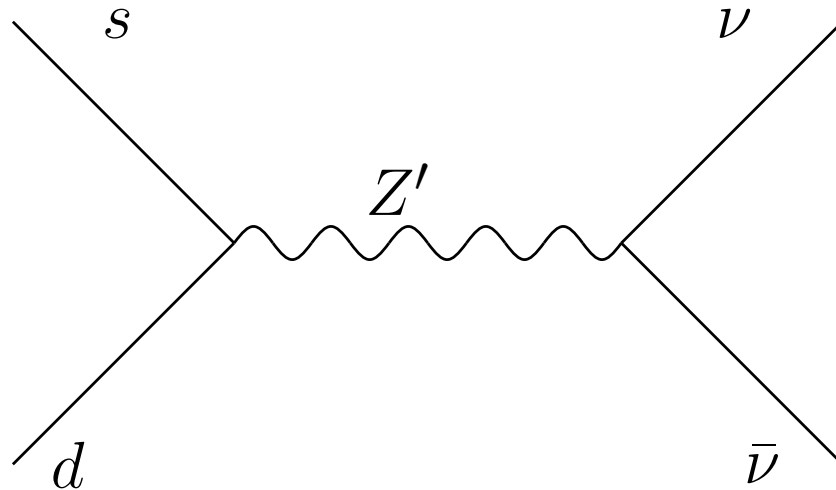
D'Ambrosio, et al.



# Non minimal new physics effects

Generic new physics can have large effect with

1. Low scale new physics
2. New flavor violation



$$\propto \frac{\kappa_{sd}^* \kappa_{\nu\nu}}{m_{Z'}^2}$$

Generically, new physics affects  $B$  and  $K$  differently



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# General properties of $K_L \rightarrow \pi \nu \bar{\nu}$

# Why is $K_L \rightarrow \pi\nu\bar{\nu}$ CP violating decay?

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- In general, three body decays do not have definite CP
- $\pi^+\pi^-$  is CP even
- $\pi^+\pi^-\pi^0$  can have both CP even and CP odd components

Is  $K_L \rightarrow \pi\nu\bar{\nu}$  a true CP violating decay?

# CP violation in $K_L \rightarrow \pi \nu \bar{\nu}$

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We neglect  $\varepsilon_K$  and

1. Consider only left handed neutrinos
  2. Assume lepton flavor conservation
  3. Neglect small  $m_K/m_W$  effects
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The only dimension 6 operator that mediate  $K_L \rightarrow \pi \nu \bar{\nu}$  is

$$O_{sd}^{ii} = \bar{s} \gamma_\mu d \times \bar{\nu}_L^i \gamma^\mu \nu_L^i$$

- This operator produces  $\pi^0 \nu_i \bar{\nu}_i$  in a CP even state
- We can think of it as  $K_L \rightarrow \pi Z^*$  two body decay
- Since  $K_L$  is CP-odd,  $K_L \rightarrow \pi \nu \bar{\nu}$  is CP violating

# CP properties of the operators

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- We choose

$$|K_L\rangle = |K\rangle + |\bar{K}\rangle$$

- The hadronic matrix element that enter  $K_L \rightarrow \pi\nu\bar{\nu}$  transform under CP as

$$\langle\pi|\bar{s}\Gamma d + \bar{d}\Gamma s|K_L\rangle \xrightarrow{CP} \langle\pi|\eta_{CP}(\Gamma)(\bar{s}\Gamma d + \bar{d}\Gamma s)|K_L\rangle$$

- Since we consider only vector interaction where  $\eta_{CP}(V) = -1$ ,  $K_L \rightarrow \pi\nu\bar{\nu}$  requires CP violation

# What is the kind of CP violation?

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In  $B$  physics we talk about three types of CPV

1. “Direct”  $|\bar{A}/A| \neq 1$
2. “Indirect”  $|q/p| \neq 1$
3. Interference between mixing and decay

$$\arg \left( \frac{q\bar{A}}{pA} \right) \neq 0$$

- $\delta_L$  is of the second type, “Indirect”
- $\mathcal{R}e(\epsilon)$  is “Indirect” and  $\mathcal{I}m(\epsilon)$  is the third type
- $\mathcal{R}e(\epsilon')$  is “direct” and  $\mathcal{I}m(\epsilon')$  is the third type
- $K_L \rightarrow \pi\nu\bar{\nu}$  is of the third type  $\Rightarrow$  the clean type

# The third type of CPV

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$$\lambda \equiv \frac{A(\bar{K} \rightarrow \pi \nu \bar{\nu})}{A(K \rightarrow \pi \nu \bar{\nu})} \frac{q}{p}$$

Then we get

$$\frac{\Gamma(K_L \rightarrow \pi \nu \bar{\nu})}{\Gamma(K_S \rightarrow \pi \nu \bar{\nu})} = \frac{1 + \lambda^2 - 2\mathcal{R}e\lambda}{1 + \lambda^2 + 2\mathcal{R}e\lambda} \xrightarrow{|\lambda|=1} \tan^2 \theta$$

where

$$\theta \equiv \arg(\lambda)$$

In the SM  $\theta \approx \beta$ , up to calculable charm contribution

# Bounding the $K_L \rightarrow \pi\nu\bar{\nu}$ rate

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$\Gamma(K_S \rightarrow \pi\nu\bar{\nu})$  may never be measured. We thus use isospin

$$\sqrt{2}A(K^0 \rightarrow \pi^0\nu\bar{\nu}) = A(K^+ \rightarrow \pi^+\nu\bar{\nu})$$

Then we get

$$R \equiv r_{is} \frac{\Gamma(K_L \rightarrow \pi\nu\bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})} = \sin^2 \theta$$

with  $r_{is} = 0.954$  takes care of isospin breaking

- $R$  measured  $\theta$  cleanly assuming only isospin
- Using  $\sin^2 \theta \leq 1$  we get a model independent bound

# More general $K_L \rightarrow \pi \nu \bar{\nu}$

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We can have CP conserving contribution once we relax each of the above mentioned assumptions

1. Consider also right handed neutrinos
2. Allow for lepton flavor violation
3. Include small  $m_K/m_W$  effects



# Right handed neutrinos

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Consider right-handed neutrinos. Then there are new dimension 6 scalar and tensor operators. For example

$$(\bar{s} \Gamma_S d) \times \bar{\nu}_R \Gamma_S \nu_L$$

- $\Gamma_S$  is a scalar operator
- Under CP
$$\Gamma_S \xrightarrow{CP} \Gamma_S \quad \Gamma_V \xrightarrow{CP} -\Gamma_V$$
- Scalar operators generate  $K_L \rightarrow \pi \nu \bar{\nu}$  in the CP limit, with one LH and one RH neutrino (SM: both are LH)
- The effect due to neutrino masses is tiny
- The spectrum is different from the standard spectrum

# $m_K/m_W$ effects

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Under CP

$$\nu(k_1)\bar{\nu}(k_2) \xrightarrow{CP} \nu(k_2)\bar{\nu}(k_1)$$

- For a CP eigenstate the  $\nu$  and  $\bar{\nu}$  must have the same spectrum
- The standard dimension 6 operator generates a symmetric spectrum
- Dimension 8 operators do not necessarily generate a symmetric spectrum
- A general spectrum is a sum of a symmetric and an antisymmetric spectrum. That is, CP-even and CP-odd
- In the SM this operator comes from the box diagram as long as we keep the external momenta

# Lepton number violation

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Consider a CP conserving new physics model where

$$A(K \rightarrow \pi \nu_i \bar{\nu}_j) \neq 0 \quad A(K \rightarrow \pi \nu_j \bar{\nu}_i) = 0$$

Due to CP

$$A(\bar{K} \rightarrow \pi \nu_i \bar{\nu}_j) = 0 \quad A(\bar{K} \rightarrow \pi \nu_j \bar{\nu}_i) \neq 0$$

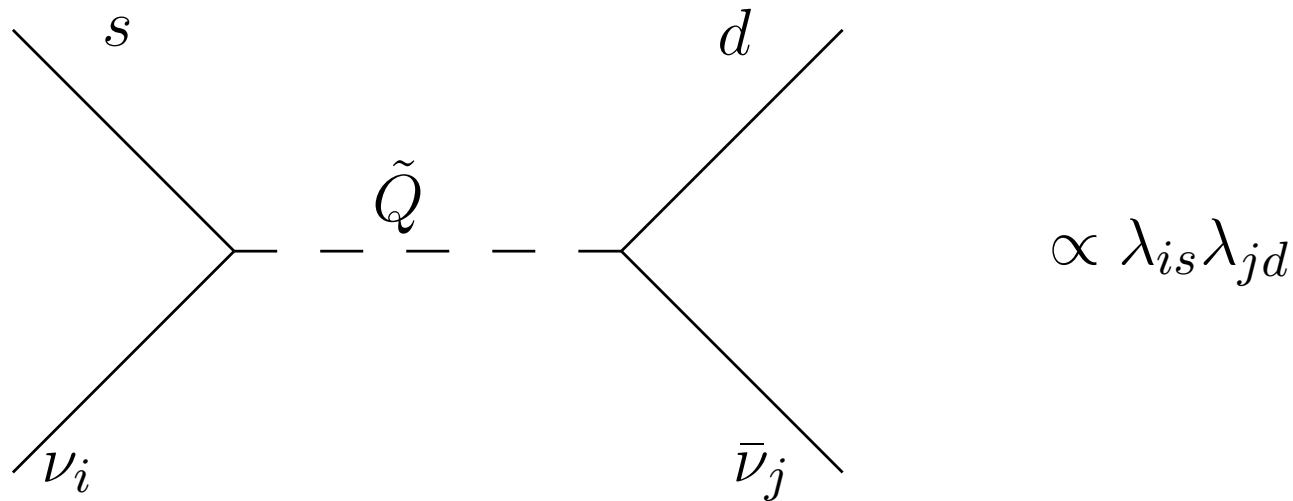
Then

$$\sqrt{2} A(K_L \rightarrow \pi \nu_i \bar{\nu}_j) = A(K \rightarrow \pi \nu_i \bar{\nu}_j) + A(\bar{K} \rightarrow \pi \nu_i \bar{\nu}_j) \neq 0$$

- In the standard CP conserving case,  $A(K_L \rightarrow \pi \nu \bar{\nu}) = 0$  since  $K$  and  $\bar{K}$  cancel each other
- This cancelation does not occur once these decay amplitudes have different magnitudes

# Example: SUSY without R-parity

We get an effective leptoquark interaction:  $\lambda_{ik}\bar{L}_i Q_k S$



• Then

$$A(K_L \rightarrow \pi \nu_i \bar{\nu}_j) \propto (\lambda_{is}\lambda_{jd} - \lambda_{id}\lambda_{js})$$

which, in general, is finite in the CP limit

# Lepton number violation: mass matrices

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We know that lepton number is violated by neutrino mass

Q: Can it generate CP conserving contribution to  $K_L \rightarrow \pi \nu \bar{\nu}$ ?

A: Practicality no. We can rotate to interaction basis

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Can the CP violating contribution be sensitive to the mixing matrix?

- Neutrino mixing: The effects are proportional to  $m_\nu$
- Sneutrino mixing: The  $K_L \rightarrow \pi \nu \bar{\nu}$  rate depends on the sneutrino masses but not their mixing angles
- Sneutrino and Slepton mixing. The rate depends on the product of the two rotation matrices

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# Conclusions

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- The  $K \rightarrow \pi \nu \bar{\nu}$  decays are very clean and interesting
- It would be nice to know their rates
- Even in the era when flavor physics is dominated by  $B$ 's, kaons are important